THEME: ROOTS OF QUADRATIC EQUATION

WEEK: 2

CLASS: SS 2

SUBJECT: FURTHER MATHEMATICS

UNIT TOPIC: ROOTS OF QUADRATIC EQUATION

LESSON TOPIC: CONDITIONS FOR QUADRATIC EQUATION

SPECIFIC OBJECTIVES: At the end of the lesson, students should be able to;

i. Define quadratic equation;
ii. State and apply conditions for quadratic equation;
iii. Find the sum and product of roots of quadratic equations;
iv. Find the quadratic given sum and product of roots

INSTRUCTIONAL RESOURCES: Charts of general solution of quadratic equation etc.

LESSON PRESENTATION: The teacher present lesson step by step by first asking the students questions based on previous lessons, for example, what is an equation? What is quadratic equation? Etc

STEP 1

MODE: Entire Class

TEACHER'S ACTIVITIES

ROOTS OF QUADRATIC EQUATION

QUADRATIC EQUATION

Definition: A quadratic equation is an equation that can be written as $ax^2 + bx + c = 0; a \neq 0$ where a, b and c are constants

Examples of quadratic equations are;

a. $3x^2 - 55x - 4 = 0$
b. $\frac{7}{5}y^2 + 7y + 22 = 0$
c. $t(t - 7) = 0$
d. $(x - 6)(x + 6) = 0$
e. $(p - 8)^2 = 0$
f. $\frac{4}{5}x^2 - \frac{3}{10}x - \frac{5}{21} = 0$

The followings are not quadratic equation;

a. $\frac{3}{x^2} + 7x - 1 = 0$
b. $5y^2 - 7\sqrt{y} + 6 = 0$
c. $11t^2 + \frac{4}{t} + \frac{7}{11} = 0$

d. $9\sqrt{m^2} + 6m - 7 = 0$

STUDENTS ACTIVITIES

1. Give five examples of quadratic equation
2. Give five examples non quadratic equation

STEP II

Exploration; fact find about the lesson objectives using the resources around

MODE: ENTIRE CLASS

TEACHER'S ACTIVITIES

DISCRIMINATE

Definition: Discriminate is a number that can be calculated from any quadratic equation. It is usually denoted by;

$$D = b^2 - 4ac$$

Where a is coefficient of x^2 , b is coefficient of x and c is a constant from any quadratic equation

Example: Determine discriminate of $3x^2 + 9x + 5 = 0$

Solution

$$a = 3, b = 9 \ and \ c = 5$$

$$D = b^2 - 4ac$$

$$= 9^2 - 4(3)(5)$$

$$= 81 - 60$$

$$= 21$$

STUDENTS ACTIVITIES

Identify quadratic equation and determine it's discriminate from the following equations;

a. $4x^2 - 55x - 4 = 0$
b. $\frac{7}{5}y^2 + 7y + 11 = 0$
c. $\frac{3}{x^2} + 7x + 1 = 0$
d. $5y^2 - 8\sqrt{y} + 6 = 0$
e. $11t^2 + \frac{4}{t} + \frac{1}{11} = 0$
f. $9\sqrt{m^2} + 6m + 7 = 0$

g. $t(t + 7) = 0$
h. $(x - 8)(x + 8) = 0$

i. $(p+8)^2 = 0$
j. $\frac{4}{3}x^2 - \frac{3}{5}x - \frac{5}{2} = 0$

STEP III

Discussion of condition about a quadratic equation

MODE: ENTIRE CLASS

TEACHER'S ACTIVITIES

CONDITIONS FOR QUADRATIC EQUATION

The discriminate provides critical/information regarding the nature of the roots/solutions of any quadratic equations

The discriminate provides the following information (Conditions) about a quadratic equation;

- ❖ If the solution is unique (one) solution/root or two different solutions/roots
- ❖ If the solutions/roots are real or imaginary(complex)
- ❖ If the solutions/roots are rational or irrational

POSITIVE DISCRIMINATE

a. If $b^2 - 4ac > 0$ and it is perfect square, then, the roots are;

- Two real roots(solutions)
- The roots are rational

Example: $\boldsymbol{x^2 + 4x - 5 = 0}$

$$a = 1, b = 4\ and\ c = -5$$

$$D = b^2 - 4ac$$

$$= b^2 - 4(1)(-5)$$

$$= 16 + 30$$

$$= 36$$

Since the discriminate is positive and a perfect square, there are two real solutions that are rational

$$x^2 + 4x - 5 = 0$$

$$(x+5)(x-1) = 0$$

$$x = -5\ or\ x = 1$$

b. If $b^2 - 4ac > 0$ and is not a perfect square, then, the roots are;

- Two real roots(solutions)
- The roots are irrational

Example: $3x^2 - 5x + 1 = 0$

$$a = 3, b = -5 \ and \ c = 1$$

$$D = b^2 - 4ac$$

$$= (-5)^2 - 4(3)(1)$$

$$= 25 - 12$$

$$= 13$$

Since the discriminate is positive and not a perfect square, then there are two real solutions (roots) that are irrational

$$3x^2 - 5x + 1 = 0$$

$$a = 3, b = -5 \ and \ c = 1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(1)}}{2 \times 3}$$

$$= \frac{5 \pm \sqrt{13}}{6}$$

$$= \frac{5 + \sqrt{13}}{6} \ or \ \frac{5 - \sqrt{13}}{6}$$

NEGATIVE DISCRIMINATE

a. If $b^2 - 4ac < 0$ and is perfect square, then, the roots are;

- No real solution/roots [two complex (imaginary) solutions/roots]
- The roots(solutions) are rational

Example: $x^2 - 4x + 5 = 0$

$$a = 1, b = -4 \ and \ \ c = 5$$

$$D = b^2 - 4ac$$

$$= (-4)^2 - 4(1)(5)$$

$$= 16 - 20$$

$$= -4$$

Since the discriminate is negative and a perfect square, there are two imaginary roots that are rational

That is

$$x^2 - 4x + 5 = 0$$

$$a = 1, b = -4 \ and \ c = 5$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{-4}}{2 \times 1}$$

$$= \frac{4 \pm i\sqrt{4}}{2}$$

$$= \frac{4 \pm 2i}{2}$$

$$= 2 + i \; or \; 2 - i$$

$\boldsymbol{b}$. If $b^2 - 4ac < 0$ and it is not a perfect square, then, the roots are;

- No real roots/solutions (two imaginary roots/solutions)
- Irrational roots/solutions

Example: $\boldsymbol{x^2 + 3x + 7 = 0}$

$$a = 1, b = 3 \; and \; c = 7$$

$$D = b^2 - 4ac$$

$$= 3^2 - 4(1)(7)$$

$$= 9 - 28$$

$$= -19$$

Since the discriminate is negative and not a perfect square, then, the two roots/solutions are irrational and complex

That is

$\boldsymbol{x^2 + 3x + 7 = 0}$

$$a = 1, b = 3 \; and \; c = 7$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-3 \pm \sqrt{-19}}{2}$$

$$= \frac{-3 \pm i\sqrt{19}}{2}$$

$$x = \frac{-3 + i\sqrt{19}}{2} \; or \; x = \frac{-3 - i\sqrt{19}}{2}$$

ZERO DISCRIMINATE

If $b^2 - 4ac = 0$, then, the root/solution is unique (one)

Example: $x^2 - 2x + 1 = 0$

$$a = 1, b = -2 \; and \; c = 1$$

$$D = b^2 - 4ac$$

$$= (-2)^2 - 4(1)(1)$$

$$= 4 - 4$$

$$= 0$$

Since the discriminate is zero, then, the solution/root is rational unique (one)

That is,

$$x^2 - 2x + 1 = 0$$

$$x^2 - x - x + 1 = 0$$

$$(x - 1)(x - 1) = 0$$

$$x = 1 \; twice$$

STUDENTS ACTIVITIES

Without solve the following equations, find the information (condition) about them

$(i)\; 2x^2 + 8x - 10 = 0 \; (ii)\; 6x^2 - 10x + 2 = 0 \; (iii)\; 2x^2 - 4x + 2 = 0 \; (iv)\; 2x^2 - 8x + 10 = 0$

$(v)\; 2x^2 + 6x + 14 = 0 \; (vi)\; 3x^2 + 9x + 21 = 0$

STEP IV

Evaluation: Teacher will evaluate students through questions relevant to the lesson objectives, for example, what is discriminate? What is the formula for discriminate? Etc.

ASSIGNMENT

i. What will be the value of P so that the quadratic equation $Px^2 - 4x + 1 = 0$ has two equal roots?
ii. Find the value of the constant K for which the equation $2x^2 + (K + 3)x + 2K = 0$ has equal roots
iii. If the roots of $(x - 1)(x - 2) = K$are equal, find the value of K
iv. Find the values of M which make the quadratic function $x^2 + 2(M + 1)x + M + 3$ a perfect square
v. What must be added to the expression $x^2 - 18x$ to make it a perfect square?
vi. If the quadratic equation $3x^2 + 7x + C$ is a perfect square, find C

REFRENCES

Further mathematics for SSS by P.N Lassa & S.A Ilori

Exam focus mathematics page 199 to 203

New further mathematics project 2

Hidden facts in further mathematics

THEME: ROOTS OF QUADRATIC EQUATION

CLASS: SS 2

WEEK: 2

SUBJECT: FURTHER MATHEMATICS

UNIT TOPIC: ROOTS OF QUADRATIC EQUATION

LESSON TOPIC: CONDITIONS FOR LINE TO INTERSECT, SUM AND PRODUCT OF ROOTS OF QUADRATIC EQUATIONS

SPECIFIC OBJECTIVES: At the end of the lesson, students should be able to;

i. State and explain condition for line to intersect the curve or not to intersect to the curve;
ii. State and apply conditions for quadratic equation;
iii. Find the sum and product of roots of quadratic equations;
iv. Find the quadratic given sum and product of roots;
v. Solve problems on roots of quadratic equations

INSTRUCTIONAL RESOURCES: Charts showing condition for line to intersect curve and not to intersect etc.

LESSON PRESENTATION: The teacher present lesson step by step by first asking the students questions based on previous lessons, for example, condition for real roots, imaginary roots and equal roots Etc

STEP 1

MODE: Entire Class

TEACHER'S ACTIVITIES

CONDITION FOR LINE TO INTERSECT, NOT TO INTERSECT AND TANGENT TO THE CURVE

a. If $b^2 - 4ac > 0$; the graph crosses the x-axis (intersect curve)

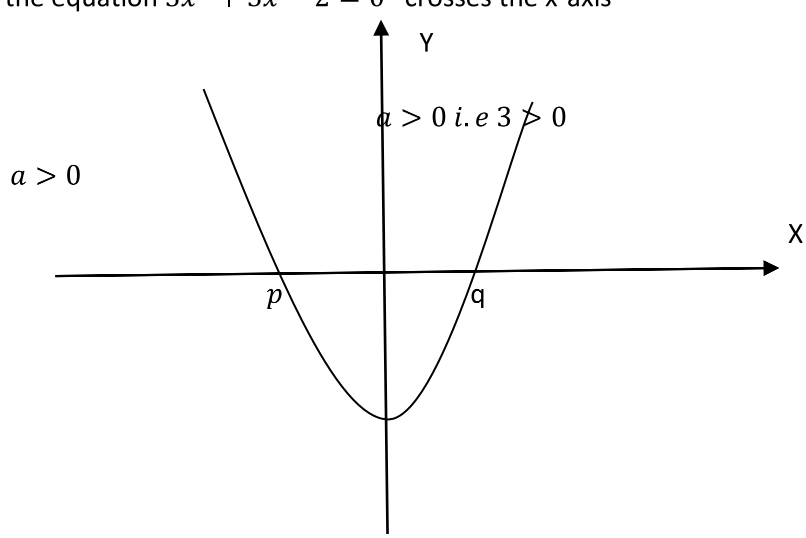
Example: $3x^2 + 5x - 2 = 0$

$$b^2 - 4ac = 25 + 24$$

$$= 49 > 0$$

The graph of the equation $3x^2 + 5x - 2 = 0$ crosses the x-axis

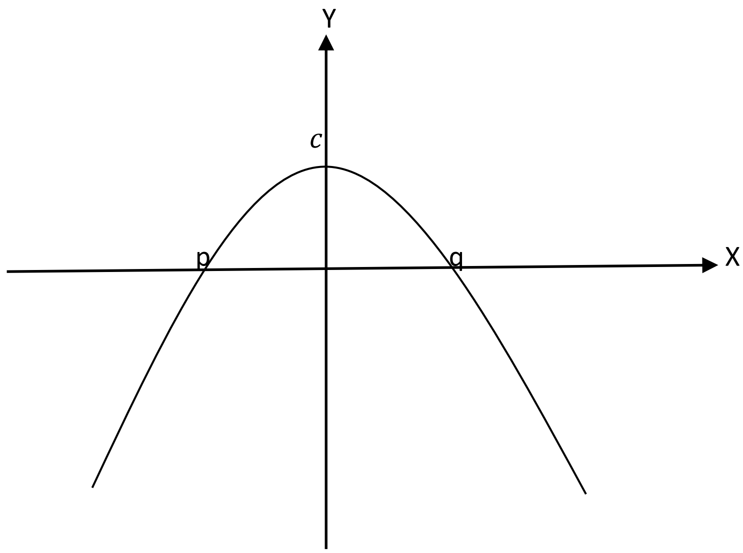
Since



c

If

$a < 0$



b. If $b^2 - 4ac < 0$, the graph is either wholly above or wholly below the x-axis

bi. If $f(0) > 0$, the graph lies wholly above x-axis

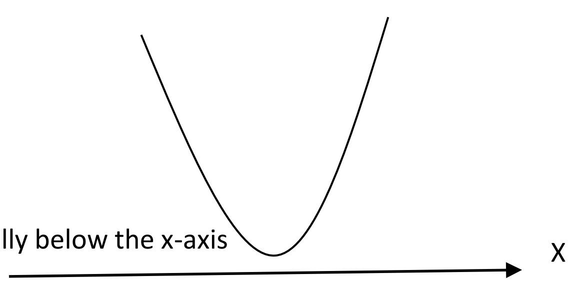
Example: $2x^2 + 5x + 4 = 0$

$$b^2 - 4ac$$

$$= 25 - 32 < 0 \ and$$

$$f(0) = 4 > 0$$

The graph lies wholly above x-axis



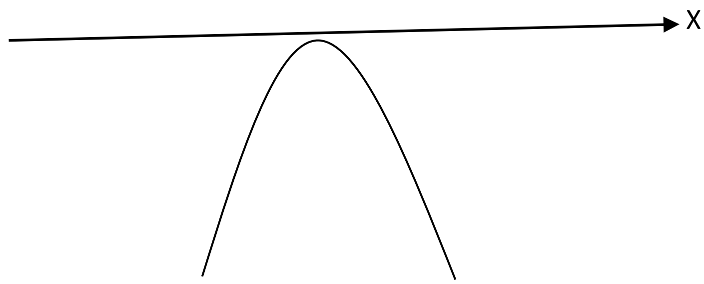
bii. If $f(0) < 0$, the graph lies wholly below the x-axis

Example: $-4x^2 + 2x - 9$

$$b^2 - 4ac$$

$$= 4 - 144 < 0 \ and\ f(0) = -9 < 0$$

The graph lies wholly below x-axis



c. If $b^2 - 4ac = 0$, the graph is tangent to x-axis

Example: $4x^2 - 20x + 25 = 0$

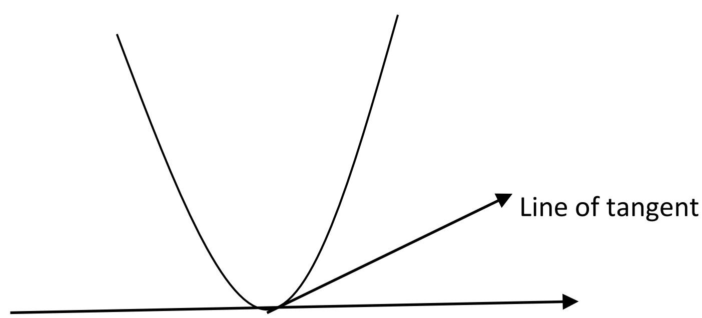
$$b^2 - 4ac$$

$$= 400 - 400$$

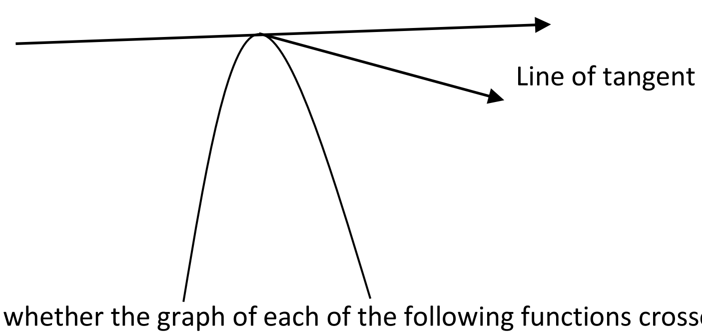
$$= 0$$

The graph is tangent to the x-axis

Since $a > 0$,



If $a < 0$,



STUDENTS ACTIVITIES

Without sketching graph, state whether the graph of each of the following functions crosses the x-axis, lies wholly above or below or tangent to the curve

$a. 6x^2 + 10x - 4 = 0$ $b. 4x^2 + 10x + 8 = 0$ $c. -8x^2 + 4x - 18 = 0$ $d. 8x^2 - 40x + 50 = 0$

STEP II

Exploration; fact find about the lesson objectives using the resources around

MODE: ENTIRE CLASS

TEACHER'S ACTIVITIES

SUM AND PRODUCT OF ROOTS

Suppose α and β are the roots of $ax^2 + bx + c = 0; a \neq 0$

Then,

$$\alpha, \beta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Let $Q = b^2 - 4ac$

$$\alpha, \beta = \frac{-b \pm \sqrt{Q}}{2a}$$

Then,

$$\alpha = \frac{-b + \sqrt{Q}}{2a} \; and \; \beta = \frac{-b - \sqrt{Q}}{2a}$$

SUM OF THE ROOTS

$$\alpha + \beta = \frac{-b + \sqrt{Q}}{2a} - \frac{b - \sqrt{Q}}{2a}$$

$$= \frac{-b + \sqrt{Q} - b - \sqrt{Q}}{2a}$$

$$= \frac{-2b}{2a} = \frac{-b}{a}$$

That is,

$$\alpha + \beta = -\frac{b}{a}$$

PRODUCT OF THE ROOTS

$$\alpha\beta = \left(\frac{-b + \sqrt{Q}}{2a}\right) \left(\frac{-b - \sqrt{Q}}{2a}\right)$$

$$= \frac{b^2 + b\sqrt{Q} - b\sqrt{Q} - Q}{4a^2}$$

$$= \frac{b^2 - Q}{4a^2}$$

$$= \frac{b^2 - (b^2 - 4ac)}{4a^2} \ since\ Q = b^2 - 4ac$$

$$= \frac{c}{a}$$

That is

$$\alpha\beta = \frac{c}{a}$$

Example: Find the sum and product of the roots of the quadratic equation $x^2 + 5x - 8 = 0$

Solution

$$a = 1, b = 5\ and\ c = -8$$

SUM

$$\alpha + \beta = -\frac{b}{a}$$

$$= -\frac{5}{1}$$

$$= -5$$

PRODUCT

$$\alpha\beta = \frac{c}{a}$$

$$= \frac{-8}{1}$$

$$= -8$$

STUDENTS ACTIVITIES

Find the sum and the product of the roots of the following quadratic equations

(i) $21x^2 - 7x + 7 = 0$ (ii) $8x^2 - x - 2 = 0$ (iii) $6y^2 + 2y + 3 = 0$ (iv) $5 - 10x - 3x^2 = 0$

(v) $4 + 2m - 4m^2 = 0$ (vi) $3 - 6p - p^2 = 0$ (vii) $4x^2 - 4\sqrt{3}x + 3 = 0$ $(viii)$ $x^2 - x = 6$

STEP III

Discussion of some useful symmetric identities

MODE: ENTIRE CLASS

TEACHER'S ACTIVITIES

SOME USEFUL SYMMETRIC IDENTITIES

A.

I. $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
II. $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$
III. $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2 = [(\alpha + \beta)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2$
IV. $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$
V. $\alpha^2\beta^2 = (\alpha\beta)^2$
VI. $\alpha^3\beta^3 = (\alpha\beta)^3$
VII. $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$

B. If the roots of a quadratic equation are $\alpha \; and \; \beta$, then the quadratic equation is

$$x^2 - (sum \; of \; roots)x + product \; of \; roots = 0$$

Example: If $\alpha \; and \; \beta$ are the roots of the equation $5x^2 - 11x + 4 = 0$, obtain the equation whose roots are;

$(i)\; \alpha + 1 \; and \; \beta + 1 \; (ii)\; \alpha - 3 \; and \; \beta - 3 \; (iii)\; 2\alpha \; and \; 2\beta \; (iv)\; 3\alpha \; and \; 3\beta \; (v)\; \frac{1}{\alpha} \; and \; \frac{1}{\beta}$

Solution

$$a = 5, b = -11 \; and \; c = 4$$

$$\alpha + \beta = -\frac{b}{a} = -\frac{(-11)}{5} = \frac{11}{5}$$

$$\alpha\beta = \frac{c}{a} = \frac{4}{5}$$

Sum of the roots

$$(\alpha + 1\,) + (\,\beta + 1)$$

$$= \alpha + \beta + 2$$

$$= \frac{11}{5} + 2$$

$$= \frac{21}{5}$$

Product of the roots

$$(\alpha + 1)\,(\beta + 1)$$

$$= \alpha + \beta + \alpha\beta + 1$$

$$= \frac{11}{5} + \frac{4}{5} + 1$$

$$= \frac{11 + 4 + 5}{5}$$

$$= \frac{20}{5}$$

$$= 4$$

Required equation

$$x^2 - (sum\ of\ roots)x + product\ of\ roots = 0$$

$$x^2 - \frac{21}{5}x + 4 = 0$$

$$5x^2 - 21x + 20 = 0$$

STUDENTS ACTIVITIES

1. If $\alpha\ and\ \beta$ are the roots of the equation $2x^2 + 7x + 3 = 0$, obtain the equation whose roots are;

$(i)\ \alpha^2\ and\ \beta^2\ (ii)\ \alpha - \beta\ and\ \alpha - \beta\ (iii)\ 2\alpha + \beta and\ 2\beta + \alpha\ (iv)\ \frac{\alpha}{\beta} and\ \frac{\beta}{\alpha}\ (v)\ \frac{1}{\alpha^2}\ and\ \frac{1}{\beta^2}$

$(vi)\ 2\alpha - \beta\ and\ 2\beta - \alpha\ (vii)\ \frac{1}{\alpha} + \frac{1}{\beta}\ and\ \frac{1}{\alpha\beta}$

2. If $\alpha\ and\ \beta$ are the roots of the equation $3x^2 - 2x - 6 = 0$, Find the equation whose roots are;

$(i)\ \alpha^3\ and\ \beta\ ^3 (ii)\ \alpha\ ^4 and\ \beta^4\ (iii)\ \frac{1}{\alpha^4}\ and\ \frac{1}{\beta^4}\ (iv)\ \frac{1}{3\alpha}\ and\ \frac{1}{3\beta}\ (v)\ \frac{1}{\alpha}\ and\ \frac{1}{\beta}$

3. If $\alpha\ and\ \beta$ are the roots of the equation $2x^2 + 6x + c = 0$, Find the value of c if;

$(i)\ \alpha = \beta\ (ii)\ \alpha = \beta + 2\ (iii)\ \frac{1}{\alpha} + \frac{1}{\beta} = -3$

STEP IV

Evaluation: Teacher will evaluate students through questions relevant to the lesson objectives, for example, the identities of $\alpha^3 +\ \beta\ ^3\ and\ \alpha\ ^4 +\ \beta^4$ Etc.

ASSIGNMENT

If $\alpha\ and\ \beta$ are the roots of the equation $x^2 - 2x - 5 = 0$, Find the equation whose roots are;

$(i)\ \alpha^2\beta\ and\ \alpha\beta\ ^2 (ii)\ \alpha\ ^2 and\ \beta^2\ (iii)\ Value\ of\ \frac{1}{\alpha\ ^2} and\ \frac{1}{\beta^2}\ (iv)\ Value\ of\ \alpha - \beta$

$(v)\ \alpha^2 + \frac{1}{\beta}\ and\ \frac{1}{\alpha} + \beta^2$

REFRENCES

Further mathematics for SSS by P.N Lassa & S.A Ilori page 70

Exam focus mathematics page 199

New further mathematics project 2

Hidden facts in further mathematics

THEME: ROOTS OF QUADRATIC EQUATION

CLASS: SS 2

WEEK: 2

SUBJECT: FURTHER MATHEMATICS

UNIT TOPIC: ROOTS OF QUADRATIC EQUATION

LESSON TOPIC: CUBIC EQUATIONS

SPECIFIC OBJECTIVES: At the end of the lesson, students should be able to;

i. Find roots of cubic equations;
ii. State symmetric identities of cubic equations;
iii. Find sum of roots of cubic equations;
iv. Find the sum and product of roots of cubic equations;
v. Find the product of roots of cubic equations;
vi. Form a cubic equation;
vii. Solve problems on cubic equations

INSTRUCTIONAL RESOURCES: Charts showing example of a cubic equation etc.

LESSON PRESENTATION: The teacher present lesson step by step by first asking the students questions based on previous lessons, for example, condition for real roots, imaginary roots and equal roots Etc

STEP 1

MODE: Entire Class

TEACHER'S ACTIVITIES

CUBIC EQUATION

Definition: Cubic equation is an equation of form $ax^3 + bx^2 + cx + d = 0, a \neq 0$ where a, b, c and d are constants

Suppose $\alpha, \beta\ and\ \gamma$ are the roots of cubic equation $ax^3 + bx^2 + cx + d = 0, a \neq 0$, then,

a. The sum of roots is given by $\alpha + \beta + \gamma = -\frac{b}{a}$
b. The sum of product of roots taking two at a time is given by $\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$
c. The product of roots is given by $\alpha\beta\gamma = -\frac{d}{a}$

The properties above are obtained by comparing the coefficients of the identity

$$ax^3 + bx^2 + cx + d = a(x - \alpha)(x - \beta)(x - \gamma)$$

SYMMETRIC IDENTITIES IN $\alpha, \beta\ AND\ \gamma$

I. $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} \equiv \frac{\alpha\beta+\alpha\gamma+\beta\gamma}{\alpha\beta\gamma}$
II. $\alpha^2\beta^2\gamma^2 \equiv (\alpha\beta\gamma)^2$

III. $\alpha^3\beta^3\gamma^3 \equiv (\alpha\beta\gamma)^3$

IV. $(\alpha+\beta)(\alpha+\gamma)(\beta+\gamma) \equiv (\alpha+\beta+\gamma)(\alpha\beta+\alpha\gamma+\beta\gamma) - \alpha\beta\gamma$

V. $\alpha^2+\beta^2+\gamma^2 \equiv (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\alpha\gamma+\beta\gamma)$

VI. $(\alpha+\beta)^2+(\alpha+\gamma)^2+(\beta+\gamma)^2 \equiv 2(\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\alpha\gamma+\beta\gamma)$

VII. $\alpha^2\beta^2+\alpha^2\gamma^2+\beta^2\gamma^2 = (\alpha\beta+\alpha\gamma+\beta\gamma)^2 - 2\alpha\beta\gamma(\alpha+\beta+\gamma)$

VIII. $\alpha^3+\beta^3+\gamma^3 \equiv (\alpha+\beta+\gamma)^3 - 3(\alpha+\beta+\gamma)(\alpha\beta+\alpha\gamma+\beta\gamma) + 3\alpha\beta\gamma$

IX. $\alpha^3\beta^3+\alpha^3\gamma^3+\beta^3\gamma^3 = (\alpha\beta+\alpha\gamma+\beta\gamma)^3 - 3\alpha\beta\gamma(\alpha+\beta+\gamma)(\alpha\beta+\alpha\gamma+\beta\gamma) + 3(\alpha\beta\gamma)^2$

If the roots of a cubic equation are $\alpha, \beta\ and\ \gamma$, then the cubic equation is

$$x^3 - (sum\ of\ roots)x^2 + (sum\ of\ product\ of\ roots)x - product\ of\ roots = 0$$

That is

$$x^3 - (\alpha^3+\beta^3+\gamma^3)x^2 + (\alpha^3\beta^3+\alpha^3\gamma^3+\beta^3\gamma^3)x - \alpha^3\beta^3\gamma^3 = 0$$

Example: Find an equation in x whose roots are cubes of the equation $2x^3+5x^2-x-1=0$

Solution

Let $\alpha, \beta\ and\ \gamma$ be the roots of the given equation. Then,

$$\alpha+\beta+\gamma = -\frac{b}{a} = -\frac{5}{2}$$

$$\alpha\beta+\alpha\gamma+\beta\gamma = \frac{c}{a} = -\frac{1}{2}$$

$$\alpha\beta\gamma = -\frac{d}{a} = \frac{1}{2}$$

The roots of the required equation are $\alpha^3, \beta^3\ and\ \gamma^3$

$$\alpha^3+\beta^3+\gamma^3 \equiv (\alpha+\beta+\gamma)^3 - 3(\alpha+\beta+\gamma)(\alpha\beta+\alpha\gamma+\beta\gamma) + 3\alpha\beta\gamma = -\frac{143}{8}$$

$$\alpha^3\beta^3+\alpha^3\gamma^3+\beta^3\gamma^3 = (\alpha\beta+\alpha\gamma+\beta\gamma)^3 - 3\alpha\beta\gamma(\alpha+\beta+\gamma)(\alpha\beta+\alpha\gamma+\beta\gamma) + 3(\alpha\beta\gamma)^2 = -\frac{5}{4}$$

$$\alpha^3\beta^3\gamma^3 \equiv (\alpha\beta\gamma)^3 = \frac{1}{8}$$

Hence, the required equation is

$$x^3 - (\alpha^3+\beta^3+\gamma^3)x^2 + (\alpha^3\beta^3+\alpha^3\gamma^3+\beta^3\gamma^3)x - \alpha^3\beta^3\gamma^3 = 0$$

$$x^3 + \frac{143}{8}x^2 - \frac{5}{8}x - \frac{1}{8} = 0$$

$$8x^3 + 143x^2 - 5x - 1 = 0$$

STUDENTS ACTIVITIES

1. If $\alpha, \beta\ and\ \gamma$ are the roots of the equation$2x^3-3x^2-x+7=0$. Find cubic equation in x whose roots are;

$(i)\ 3\alpha, 3\beta\ and\ 3\gamma\ (ii)\ \alpha - 2, \beta - 2\ and\ \gamma - 2\ (iii)\ \alpha + \beta, \beta + \gamma\ and\ \gamma + \alpha\ (iv)\ \alpha^2, \beta^2\ and\ \gamma^2$

2. If $\alpha, \beta\ and\ \gamma$ are the roots of the equation $x^3 - 2x^2 - 3x + 4 = 0$. Find cubic equation in x whose roots are;

$a. (i)\ 2\alpha, 2\beta\ and\ 3\gamma\ (ii)\ \frac{1}{\alpha}, \frac{1}{\beta}\ and\ \frac{1}{\gamma}\ (iii)\ \alpha + \beta, \alpha + \gamma\ and\ \beta + \gamma\ (iv)\ \frac{1}{\alpha^2}, \frac{1}{\beta^2}\ and\ \frac{1}{\gamma^2}$

b. Evaluate $(\alpha + \beta)^2 + (\alpha + \gamma)^2 + (\beta + \gamma)^2$